USE OF TETROONS TO INVESTIGATE THE KINEMATICS AND DYNAMICS OF THE PLANETARY BOUNDARY LAYER

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ABSTRACT

Discussed is the use of tetroons to obtain estimates of the Reynolds stresses, the rate of production of eddy kinetic energy through these stresses, the coefficient of eddy viscosity, and the viscous dissipation within the planetary boundary layer. At an average height of about 3000 ft. (admittedly in the upper portion of the boundary layer) the tetroons yield a mean value for the zonal Reynolds stress of 1.5 dynes/cm². This value increases by a factor of three as the lapse rate increases by a factor of one-half. The tetroons yield an average value for the rate of production of eddy kinetic energy through Reynolds stresses of 5.4 cm.²/sec.³, but this value is probably an underestimate inasmuch as the tetroons appear systematically to underestimate the wind shear in the vertical. The rate of production of eddy kinetic energy through Reynolds stresses decreases radically with increase in lapse rate. On the average, both of the above parameters increase with increase in the 3000-ft wind speed.

The mean tetroon-derived value for the coefficient of eddy (dynamic) viscosity is 0.57×10^3 gm. cm.$^{-1}$ sec.$^{-1}$ This mean value appears somewhat large (in accord with the above-mentioned underestimate of the vertical shear) and the scatter of inividual flight values suggests that this may be too sensitive a parameter to estimate from the tetroon data.

Equating the rate of production of eddy kinetic energy through Reynolds stresses to the dissipation appears to yield too large a value for the dissipation. Consequently, buoyancy and flux divergence terms probably can not be neglected and it may be necessary to place temperature instruments on the tetroons before acceptable dissipation estimates can be obtained.

1. INTRODUCTION

Atmospheric processes within the planetary boundary layer (the layer from the earth's surface to the geostrophic wind level) have considerable influence upon atmospheric circulations in the free atmosphere. In fact, an appreciable improvement in forecasting might result from increased knowledge of the interplay between the planetary boundary layer and the free atmosphere. With the understanding that an accurate diagnosis must precede an accurate prognosis, let us consider techniques for increasing our knowledge of this interplay.

Above the surface boundary layer, meteorological information is obtained at present through the use of vertical soundings, moored balloons, isolated towers and masts, and intermittent airplane traverses. Conventional vertical soundings are inefficient since the layer of interest is traversed so rapidly. Furthermore, it is probably not feasible to establish a sufficiently dense network of vertical sounding stations to obtain information on the mesoscale. The latter reasoning also applies to moored balloons. Towers and masts may never be tall enough to probe the full extent of the planetary boundary layer, and in addition the establishment of a national network of such towers is unlikely because of the cost and possible aircraft hazard. Routine aircraft traverses are also unlikely for these same reasons.

This paper presents the thesis that the most logical way of obtaining meteorological information within the planetary boundary layer is through the use of horizontal soundings; that is, through the use of superpressured, constant volume balloons (tetroons). Tetroons tend to float along an isopycnic surface, but it has become apparent that they are easily displaced from their equilibrium floating surface by vertical air motions. Thus, tetroons appear to yield fairly reliable estimates of the 3-dimensional air motion, and their possible use in the investigation of the kinematics and dynamics of the planetary boundary layer should be carefully examined. In this paper we consider the use of tetroons to obtain estimates of the Reynolds stresses, the rate of production of eddy kinetic energy through Reynolds stresses, the coefficient of eddy viscosity, and the viscous dissipation. As temperature and pressure data become available from tetroon flights, many other meteorological parameters within the planetary boundary layer could be investigated.

2. ESTIMATION OF THE REYNOLDS STRESSES

There is considerable interest in the mechanism by which the angular momentum transported poleward by synoptic-scale eddies is transported downward to the earth's surface, and the rate at which this downward transport takes place in various localities and under various weather regimes. Tetroons might not yield reasonable values of the vertical momentum flux because of their tendency to return to an equilibrium floating level and because they do not present the same shape to the air when they move downward as when they move upward. Because numerous, simultaneous estimates of vertical momentum flux from tetroons and fixed point instruments at identical heights are lacking, we have no alternative in investigating the usefulness of tetroons in this regard but to see whether, in the mean, the tetroons do yield reasonable flux values.

The zonal Reynolds stress (τ_x), (equivalent to the vertical flux of momentum), is given by

$$\tau_x = -\overline{\rho u' w'} \tag{1}$$

where ρ is density, u and w are zonal and vertical velocities, and the primes indicate deviation from the mean for each tetroon flight. The second numerical column in table 1 shows the mean tetroon-derived zonal stress at locations where the tetroon vertical velocity could be obtained (by radar) with considerable accuracy. The velocity components were obtained at 1-min. intervals; the density is assumed constant. Since there should be some correlation between the zonal stress near 3000 ft. and at the earth's surface, it is encouraging, from the point of view of the representativeness of the tetroon technique, that the tetroon-derived zonal stress is very small for the over-water flights from Wallops Island (note that, over the sea, the boundary layer may not extend to 3000 ft.), fairly small for flights over the rolling hills of south central England (Cardington), and quite large for flights in the desert-mountain terrain near Yucca Flat, Nev. However, the smaller number of tetroon observations over the sea near Cape Hatteras and over the hilly terrain near Cincinnati also yields relatively large values of zonal Reynolds stress. At Cape Hatteras the winds were quite strong because of the presence of a hurricane nearby. The stress at Cincinnati is based on only two tetroon flights which were released simultaneously near noon. The fairly large value of the zonal stress derived from these two flights may be ascribed partly to a diurnal variation in the magnitude of the zonal stress, as will be discussed later in connection with figure 1. Incidentally, the values of the zonal stress derived from these two simultaneously released flights were 3.04 and 2.93 dynes/cm^2. The similarity in these two values also provides evidence for the representativeness of the tetroon technique.

TABLE 1.—*Mean tetroon-derived value of zonal Reynolds stress (τ_x) and rate of production of eddy kinetic energy through Reynolds stresses ($\dot{K}_R$) for various tetroon launch sites*

Launch site	Mean tetroon height (ft.)	τ_x (dynes/cm.^{2})	$\dot{K}_R$ (cm.^{2}/sec.^{3})	Observations
Wallops Island, Va.	2,700	0.08	−0.66	468
Las Vegas, Nev.	3,500	3.25	15.90	583
Cardington, England	2,300	.29	−.35	871
Cape Hatteras, N.C.	3,500	4.36	21.78	150
Cincinnati, Ohio	2,200	2.99	2.65	256
Average	2,800	1.55	5.41	2,328

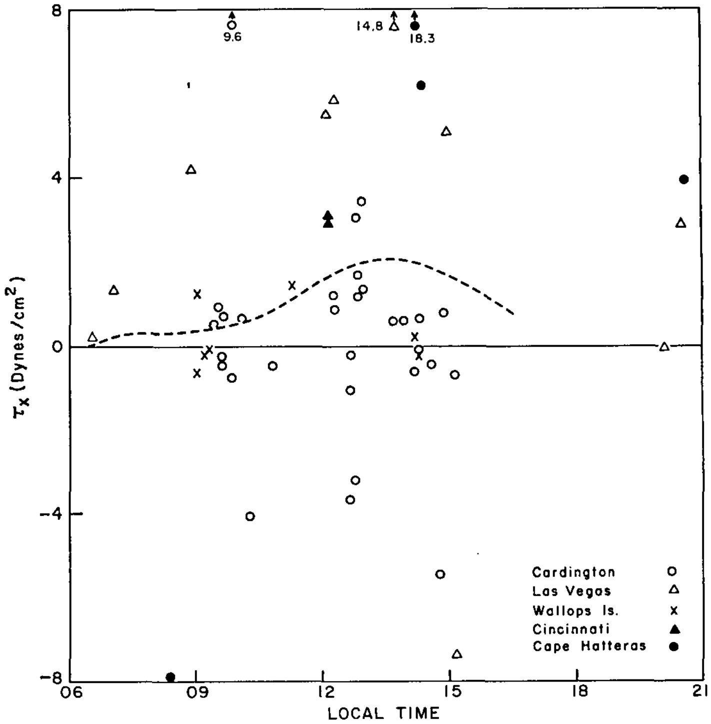

FIGURE 1.—Variation with time of day of zonal Reynolds stress (τ_x), as derived from tetroon flights at heights near 3000 ft. *The dashed line represents the smoothed average.*

If the tetroon-derived zonal stress estimates for each locality are weighted according to number of observations, a mean zonal stress of 1.55 dynes/cm.^{2} is obtained. This value refers to a mean latitude of 43° N. On the basis of the meridional, geostrophic angular momentum flux in January 1959, Mintz [1] estimated a value of 1.2 dynes/cm.^{2} for the mean zonal stress at the ground at this latitude. Of course, even if the tetroons did accurately portray the vertical momentum flux at a given time and location, it would be purely fortuitous if the distribution of tetroon launch sites was representative of the whole Northern Hemisphere at this latitude. Nevertheless, it is a hopeful portent that, as an ensemble, the tetroons do yield a reasonable approximation to the generally accepted zonal stress value of 1 dyne/cm.^{2}

Figure 1 shows the zonal Reynolds stress derived from individual tetroon flights as a function of local time. There are indications from this figure that the zonal stress is greater in the early afternoon than in the morning or late afternoon. For example, at Cardington 6 flights released shortly after noon yielded a zonal stress exceeding 1 dyne/cm.^{2}, a value not exceeded (with one exception) on any of the morning or late afternoon flights from this site. In general, the values of the zonal stress obtained at Las Vegas (Yucca Flat) also exhibit a diurnal trend. The Wallops Island and Cape Hatteras flights are not so convincing with regard to diurnal variability, partly because of the distribution of release time with time of day. The dashed line in figure 1 represents a smoothed average of

TABLE 2.—*Tetroon-derived zonal Reynolds stress* (τ_x) *and rate of production of eddy kinetic energy through Reynolds stresses* ($\dot{K}_R$) *as a function of lapse rate* (γ)

Launch site	γ (°K./100m.)	τ_x (dynes/cm.²)	$\dot{K}_R$ (cm.²/sec.³)	Observations
Wallops Island	0.30	−0.29	−0.19	245
	.63	.49	−1.17	223
Las Vegas	.65	1.64	29.70	289
	.95	4.82	2.29	294
Cardington	.72	.11	1.20	225
	.85	.81	−1.45	219
Cape Hatteras	.50	3.80	28.75	94
	.69	5.30	10.05	56
Average	.55	.92	13.50	853
	.81	2.53	.83	792

TABLE 3.—*Tetroon-derived Reynolds stress* (τ) *and rate of production of eddy kinetic energy through Reynolds stress* ($\dot{K}_R$) *as a function of wind speed* (V)

Launch site	V (m./sec.)	τ (dynes/cm.²)	$\dot{K}_R$ (cm.²/sec.³)	Observations
Wallops Island	3.8	0.23	−0.19	245
	10.9	.67	−1.17	223
Las Vegas	3.0	1.72	−.15	269
	7.5	5.90	29.60	314
Cardington	4.5	.53	−.69	495
	10.0	.19	.10	376
Cape Hatteras	7.8	6.80	28.75	94
	11.1	3.60	10.05	56
Average	4.2	1.29	2.23	1,103
	9.5	2.35	9.31	969

zonal Reynolds stress as a function of time of day as derived from all the tetroon flights. While there is a certain danger in this technique, inasmuch as the tetroon release times vary from site to site, the tetroon data available do suggest a variation in the value of the zonal stress from 2 dynes/cm.² in the early afternoon to less than 0.5 dynes/cm.² in the early morning.

The data presented in figure 1 intimate that the magnitude of the zonal Reynolds stress is proportional to lapse rate. The relation between lapse rate and zonal stress is specified in the first two numerical columns of table 2, where the tetroon flights from each launch site have been divided into two groups on the basis of lapse rate. At Cape Hatteras and Las Vegas the lapse rate was estimated from a temporal interpolation of 12-hr. radiosonde data obtained at these locations. At Wallops Island a temporal and spatial interpolation was required since there were no radiosonde ascents at Wallops Island at this time. Very accurate lapse rate data were obtained by means of instrumented barrage balloons during the time of some of the Cardington tetroon flights. Table 2 shows that at all four tetroon launch sites there is a fairly pronounced increase in zonal Reynolds stress with increase in lapse rate. On the average the zonal stress increases by nearly a factor of three as the lapse rate increases by a factor of one-half. This result is not unexpected inasmuch as during relatively unstable conditions the momentum exchange between high and low levels should be facilitated.

The stress at the earth's surface is often assumed to be proportional to the square of the wind speed at anemometer level. However, since a large surface stress implies a large downward flux of momentum at higher levels, it is not surprising that there is some correlation between wind speed and magnitude of the Reynolds stress at 3000 ft., as shown in the first two numerical columns of table 3. On the average the tetroon data suggest that, at 3000 ft., the Reynolds stress is proportional to wind speed to about the two-thirds power. The fact that there is a correlation between wind speed and lapse rate at 3000 ft. complicates the decision as to whether, basically, the Reynolds stress at this level is related to lapse rate or to wind speed.

3. ESTIMATION OF RATE OF PRODUCTION OF EDDY KINETIC ENERGY THROUGH REYNOLDS STRESSES

The rate of production of eddy kinetic energy through Reynolds stresses, $\dot{K}_R$, is given by

$$\dot{K}_R = -\overline{V'w'}\frac{\partial V}{\partial z} \tag{2}$$

where V is wind speed and w is vertical velocity. In order to estimate this quantity from tetroon data alone it obviously is necessary to estimate the vertical wind shear from tetroon data. A priori, it is not evident that the vertically oscillating tetroon will yield exactly the same wind shear as fixed-point wind instruments. For example, as the tetroon moves into a region of faster moving air it may not immediately adapt to the new wind speed and this would lead to a slight underestimate of the true shear. Alternatively, the tetroon might become embedded in a bubble or column of ascending air which did not exactly partake of the environmental air motion. The existence of a vertical momentum flux makes the latter alternative especially likely.

A comparison between tetroon-derived wind shear in the vertical and the shear derived from fixed-point instruments may be obtained from the Cardington experiments, inasmuch as at Cardington wind speed was obtained at three different levels on a barrage-balloon cable during the period of tetroon flights. The vertical wind shear was estimated from each tetroon flight by finding the mean tetroon height and then averaging the tetroon-derived wind speeds above and below this point. The resulting speed difference was assumed to apply to the height interval specified by the mid-points of the higher and lower segments. This technique, while simple and objective, has the disadvantage that speed changes along the trajectory may mask the desired speed changes with height. For example, if the tetroon is lost after starting a descent from a high portion of its trajectory, and if the speed along the trajectory (at any given level) is decreasing with time, the relatively light wind on the last crest will produce an underestimate of the actual wind shear in the vertical. Of course, the longer the tetroon flight, the smaller the resultant bias, but it is apparent from inspection that on some of the short Cardington flights this phenomenon is playing a role.

On the basis of 23 individual comparisons at Cardington, the correlation between vertical wind shear derived from tetroon flights (in the above manner) and from fixed-point wind instruments is 0.52. However, the slope of the associated regression line is only 0.31, implying that, on the average, the tetroon *does* underestimate the vertical wind shear. Thus, the values of $\dot{K}_R$ obtained from tetroon flights should, in the usual case of a wind increase with height, probably be considered conservative.

The third numerical column in table 1 presents mean values of $\dot{K}_R$ for the various tetroon launch sites. The values are quite chaotic, with negative values at Wallops Island and Cardington and large positive values at Las Vegas and Cape Hatteras. Some idea of the possible bias in $\dot{K}_R$ resulting from use of tetroon-derived wind shears is given by the data at Cardington where the mean value of $\dot{K}_R$ obtained through the use of wind shears derived from the fixed point data was 0.31 cm.^{2} sec.$^{-3}$ greater than the value obtained from the tetroon data.

In the mean for all tetroon flights, an average $\dot{K}_R$ value of 5.41 cm.^{2} sec.$^{-3}$ is obtained. This rate of kinetic energy production would, over a 24-hr. period, correspond to an increase in wind speed from 10 to 12 m. sec.$^{-1}$ at the given level. Thus, the rate of eddy kinetic energy production through Reynolds stresses indicated by tetroon flights near 3000 ft. is really quite small.

Figure 2 shows, as a function of local time, the rate of eddy kinetic energy production through Reynolds stresses as derived from individual tetroon flights. The flights from Cardington show some tendency for $\dot{K}_R$ to be positive in the morning and negative in the afternoon. The small values of $\dot{K}_R$ obtained from the Wallops Island flights show somewhat the same tendency, as do the large values obtained at Las Vegas. At Cape Hatteras, in the vicinity of the hurricane, the values of $\dot{K}_R$ were large regardless of time of day. The dashed line in figure 2 represents a smoothed average of rate of production of eddy kinetic energy through Reynolds stresses as a function of time of day as derived from all the tetroon flights. It was mentioned in connection with figure 1 that there is a certain danger in this technique, but nevertheless the

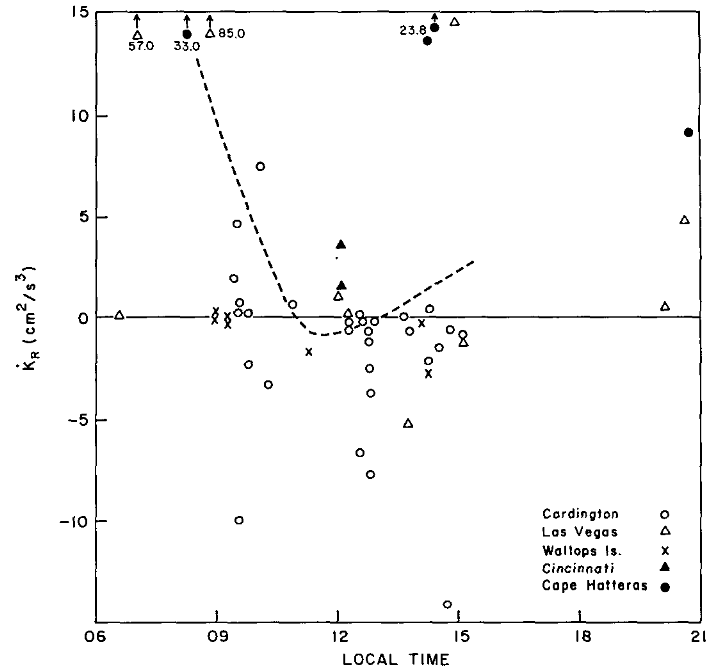

FIGURE 2.—Variation with time of day of the rate of production of eddy kinetic energy through Reynolds stresses ($\dot{K}_R$), as derived from tetroon flights at heights near 3000 ft. The dashed line represents the smoothed average.

data do suggest that $\dot{K}_R$ becomes negative near noon. This is of interest because it is just at this time of day that eddy kinetic energy is likely to be produced by buoyancy forces. More will be said about this in section 5.

Figure 2 implies that the rate of production of eddy kinetic energy through Reynolds stresses is inversely proportional to lapse rate. This is specified more precisely by the first and third numerical columns of table 2. At all launch sites there is good evidence for an increase in $\dot{K}_R$ as the lapse rate decreases. The Cardington data are particularly impressive in this regard inasmuch as excellent lapse rate data were obtained at the time of some of the tetroon flights. Table 4 shows values of $\dot{K}_R$ and lapse rate for paired morning and afternoon tetroon flights. The morning tetroon flights, made under conditions of relatively stable lapse rate, all yield positive $\dot{K}_R$ values, while none of the afternoon flights, made under conditions of relatively unstable lapse rate, yields a positive $\dot{K}_R$.

The first and third numerical columns of table 3 show that, on the average, the tetroons indicate an increase in $\dot{K}_R$ with increase in wind speed. However, this tendency is far from clear-cut, with the flights over the sea from Wallops Island and Cape Hatteras illustrating an opposite tendency.

TABLE 4.—*Tetroon-derived rate of production of eddy kinetic energy through Reynolds stresses* ($\dot{K}_R$) *for paired morning and early afternoon tetroon flights from Cardington*

Morning flight	Observations	Lapse rate (°K./100m.)	$\dot{K}_R$ (cm.^{2}/sec.^{3})	Afternoon flight	Observations	Lapse rate (°K./100m.)	$\dot{K}_R$ (cm.^{2}/sec.^{3})
5	22	0.38	4.61	6	37	0.84	−1.61
8	40	.77	1.91	9	39	.90	−.23
14	72	.74	.67	15	47	.77	0
16	46	.69	.72	17	29	.78	−7.78
24	45	.86	.21	25	67	.92	−.35
Average	225	.72	1.20	Average	219	.85	−1.45

4. ESTIMATION OF THE COEFFICIENT OF EDDY VISCOSITY

The coefficient of dynamic eddy viscosity (μ_e) represents the factor of proportionality between shearing stress and

TABLE 5.—*Mean tetroon-derived value of coefficient of dynamic eddy viscosity (μ_e) for various tetroon launch sites*

Launch site	μ_e(gm. cm.$^{-1}$ sec.$^{-1}$)	Observations
Wallops Island	-0.05×10^3	468
Las Vegas	1.19	583
Cardington	−.62	871
Cape Hatteras	3.63	150
Cincinnati	2.55	256
Average	0.57×10^3	2,328

vertical wind shear or,

$$\tau=\mu_e \frac{\partial V}{\partial z}=\rho\nu_e \frac{\partial V}{\partial z} \tag{3}$$

where ν_e represents the coefficient of kinematic eddy viscosity. Since the coefficient of eddy viscosity is the quotient of two small numbers, each of which is only imperfectly obtained from tetroon data, one would anticipate certain difficulties in obtaining reliable values of this parameter from tetroon data. In particular, since the tetroons appear to underestimate the vertical wind shear, the coefficient should be overestimated in the mean.

The coefficient of kinematic eddy viscosity may be considered an exchange coefficient and as such implies a transfer of momentum in the direction of the gradient of momentum. If the air motions are unorganized, that is, turbulent in the usual sense, the coefficients of eddy viscosity must always be positive. This is undoubtedly true near the ground, but aloft there is no reason why organized air motions could not exist which would transport momentum against the momentum gradient (the northward eddy transport of angular momentum on the south side of the subtropical jet stream is an example of this). Table 5 shows that, on the average, the tetroon flights from Cardington and Wallops Island yield such a negative value for the kinematic and dynamic viscosity. On the basis of all the flights an average value of 0.57×10^3 gm. cm.$^{-1}$ sec.$^{-1}$ is obtained for the dynamic eddy viscosity, which approximately corresponds to a value of 5.7×10^5 cm.^{2} sec.$^{-1}$ for the kinematic eddy viscosity.

Figure 3 shows values of the coefficient of dynamic eddy viscosity, derived from individual tetroon flights, as a function of local time. The dashed line indicates some tendency for the coefficient to be negative shortly before noon, somewhat in agreement with the variation of $\bar{K}_R$ noted in figure 2. As mentioned above, this implies a flux of momentum against the momentum gradient at this time of day as a result of the existence of organized vertical motion systems. The worrisome feature of figure 3 is the wide scatter of μ_e values derived from individual tetroon trajectories. When it is recalled that the average value is generally placed between 0.05 and 0.5×10^3 cm.$^{-1}$ sec.$^{-1}$, it is apparent that the tetroon flights are yielding unrealistically large values for μ_e, in accord with the underestimate of the vertical shear mentioned above. Thus, there is some evidence that the coefficient of eddy vis-

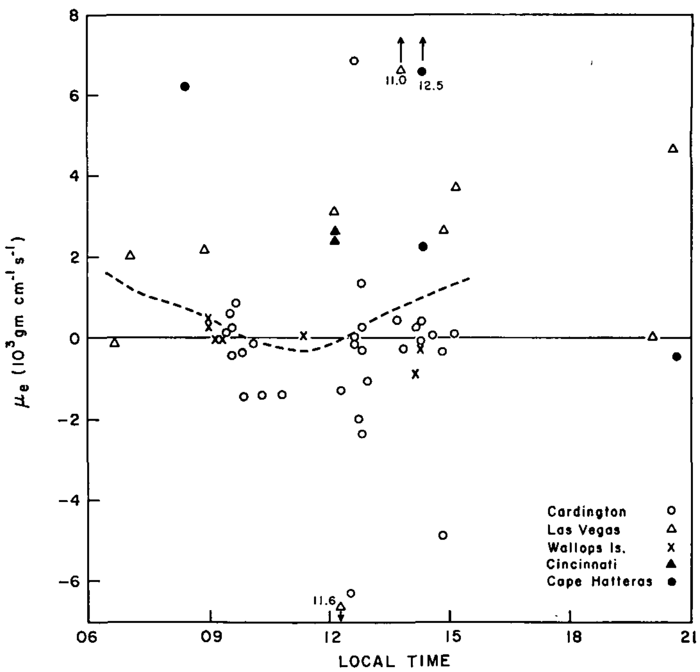

FIGURE 3.—Variation with time of day of the coefficient of dynamic eddy viscosity (μ_e) as derived from tetroon flights at heights near 3000 ft. The dashed line represents the smoothed average.

cosity is too sensitive a parameter to be derived from tetroon flights through the use of equation (3), unless longer flights positioned with great accuracy (which is now possible) obtain better and more representative sampling.

5. ESTIMATION OF THE VISCOUS DISSIPATION

Within any region of the atmosphere, there is a balance between the gain of eddy kinetic energy due to Reynolds stresses, buoyancy forces (unstable environment), and flux convergence, and the loss of eddy kinetic energy due to viscous dissipation, the work done against gravity in a stable environment, and flux divergence. This energy budget is frequently simplified by neglecting all terms except those involving the increase of eddy kinetic energy due to Reynolds stresses and the decrease of eddy kinetic energy due to viscous dissipation. Recently, there have been indications that this procedure is not as bad as one might expect, because the effect of buoyancy tends to be canceled by the effect of flux divergence. It has been suggested by Pack [2] that dissipation could be estimated from tetroon data through use of the above simplification. Thereby, one notes from table 1 that the tetroons yield a mean value for the viscous dissipation of 5.41 ergs gm.$^{-1}$ sec.$^{-1}$ While this value is nearly an order of magnitude larger than the value indicated by Wilkins [3] for the 3000-ft. level, the large variation in the value among tetroon launch sites makes any average value appear somewhat meaningless. In particular, the negative values of the dissipation derived from the Wallops Island and Cardington flights by this technique suggest that these other terms in the energy budget can not be ignored.

Table 2 showed that at all launch sites the rate of production of eddy kinetic energy due to Reynolds stresses was less when the lapse rate was large than when the lapse rate was small. This may indicate the efficacy of the buoyancy forces in producing eddy kinetic energy when the lapse rate is relatively unstable (at all launch sites except Las Vegas the neutral lapse rate may be the moist adiabatic rather than the dry adiabatic). Inasmuch as there is no apparent method by which the contribution of buoyancy forces to the production of eddy kinetic energy can be estimated from tetroon flights as now constituted, it would appear that the estimation of viscous dissipation from tetroon flights is a questionable procedure at best except, perhaps, in cases when the lapse rate is neutral and one has reason to believe that there is no convergence or divergence of the flux of eddy kinetic energy. When temperature-measuring devices become available on the tetroons, it should be possible to estimate the vertical heat flux, and hence the effect of buoyancy forces on the energy budget.

6. CONCLUSION

The data presented in this study indicate that representative values of Reynolds stress are obtained from tetroon flights. Since the tetroons can be set to float at any desired level, the variation of Reynolds stress with height may be obtained by this technique. However, since the tetroons appear to underestimate the vertical wind shear, they yield a conservative estimate of the rate of production of eddy kinetic energy through Reynolds stresses. On the other hand, inasmuch as the coefficient of eddy viscosity equals the ratio of stress to shear, the tetroons should overestimate this quantity. Moreover, this coefficient is so sensitive to values of stress and (particularly) shear, that there is some doubt that tetroons will ever yield reliable values of the coefficient of eddy viscosity. Dissipation is another parameter which is difficult to evaluate from tetroons because the effect of buoyancy and flux divergence apparently cannot be omitted from the equation expressing the balance of eddy kinetic energy. Since the buoyancy is proportional to the vertical heat flux, it should be possible to estimate this quantity from thermometer-equipped tetroons. Estimation of the flux divergence of kinetic energy from tetroon flights may be much more difficult.

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